QUANTUM COMPUTING: A REVIEW ARTICLE

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Quantum computing schemes and quantum computers have been physically implemented in the recent years. Both nations and private institutions are investing in quantum computing due to its advantages over classical computers for some advanced problems. On the one hand, quantum computers are being developed with higher number of bits, while on the other hand, experts indicate that it may not be wise to use quantum computers on all kinds of areas. This article aims to provide an overall picture of quantum computing with an historical approach and an introduction of the essentials of quantum computing. Some major challenges and big companies working on the subject are also presented to understand the obstacles and the future of quantum computing.

INTRODUCTION

From the introduction of quantum mechanics in the early $20th$ century and the concept of quantum computers in the late $20th$ century, quantum computing and the physical implementations of it (such as quantum computers) developed significantly. Although research on the physical implementations of quantum computers started in 1980s, the working quantum computers started to be seen around 2000s. Once it started, the interest about and advancement of the quantum computers grew exponentially (see Fig. [1\)](#page-0-0). Currently, there exists many companies such as Google, IBM, Microsoft, Intel, D-Wave, etc. who produced or trying to produce their own working quantum computers. Not all quantum computers work on the same technology, and each of these technologies come with their own challenges. Although it is widely accepted that quantum computers can exceed classical computers in speed and type of operations possible, it may not always wise to use quantum computers for all kinds of problems. In the light of these, the essentials of the quantum computers are presented in the next section. An overall history, a comparison between quantum computers and classical computers, some major challenges of quantum computing (and computers), companies currently working on quantum computing, and the potential areas of use of quantum computing in the near-future are presented in the following sections of this article.

QUBITS

Quantum bits, qubits, are "the fundamental units of quantum information" [\[2\]](#page-6-0). To understand qubits, comparing them with classical bits that are used in the classical computers can be helpful. Classical computers use bits to encode information. Physically, bits correspond to transistors inside the computer, which can be switched ON/OFF. Therefore, bits take values of 1 (ON), or 0

FIG. 1: Timeline chart of the quantum hardware development, indicating its accelerate in the last years[\[1\]](#page-6-1)

(OFF). [\[3\]](#page-6-2). Moreover, each bit can be switched on, or off independent of the other bits. On the contrary, quantum bits cannot be controlled independently, and capable of representing more than only 0 and 1. These properties of qubits are mainly due to the two principles of the quantum mechanics, namely superposition and entanglement.

Superposition of Qubits

A qubit can be in multiple states at once. Traditional bits can only take values 0 or 1, while quantum bits are able to be in a state between 0 and 1. The state of a qubit is represented by a Bloch sphere, which is the spherical representation at the right of the Fig. [2.](#page-1-0) While the states of bits are discrete (0 or 1), the possible states of qubits range from 0 to 1, distinguished from each other by different phases. A measured qubit has one of the states represented on the sphere, but the state of a qubit is unknown until it is measured.

Entanglement of Qubits

Unlike bits -separate transistors inside classical computers- qubits are related to each other in a way that

FIG. 2: Bit and Qubit Representations [\[3\]](#page-6-2)

prevents them acting independent of each other. Entangled qubits are linked with a state (such as spin) that is common to each other. If one uses spin states of two entangled electrons as two qubits and measures the spin of one of the electrons, the spin state of the other electron is not free to have any values. In other words, the spin state of the second electron (qubit) is determined that prevents it from having any arbitrary state independent of the first electron's spin state (qubit).

Generationg Qubits

Qubits, as described above, are mainly elementary elements of the quantum computers to store information. Superposition and entanglement are the two basic properties of qubits that are exploited to have quantum computers. It is noted that transistor states are used to implement bits in classical computers. In quantum computers, the question is then what is to be used to represent quantum bits.

Since the quantum mechanical properties are to be exploited to create quantum computers, it is required to use quantum mechanical building blocks (like photons, protons or electrons) to create a qubits [\[3\]](#page-6-2). Thus, it was (is) a challenge to generate and make them preserve their states. For example, generating qubits require accessing and controlling the quantum properties of electrons with light or magnetic fields, if the states of electrons are used as qubits. It is mostly the spin property of individual electrons that are being controlled, and applying light, magnetic fields, or electromagnetic pulses are the mainly used methods to control them [\[4\]](#page-6-3). In fact, the question of what is to be used as qubits has been an important question, and some of the answers to it is presented in the following section.

PHYSICAL IMPLEMENTATIONS OF QUANTUM COMPUTING

Below, six of the methods to generate qubits (or to physically implement quantum computers) are presented briefly. Although there exists many more approaches,

these six methods are the widely encountered ones and important to clarify the concept of qubits.

Nuclear Magnetic Resonance

Being (one of) the first method to achieve a working quantum computer (of 2 qubits) [\[5\]](#page-6-4), nuclear magnetic resonance (NMR) quantum computers are important in the history of quantum computing. It uses the spin states of nuclei within molecules as qubits [\[6\]](#page-6-5). The sample can be in liquid state, or solid state. Magnetic field and radio frequency pulses are used to control the qubits (molecules). It can operate at room temperature [\[7\]](#page-6-6). This technology is used by the companies IBM and SpinQ [\[8\]](#page-6-7)[\[9\]](#page-6-8).

Spin Qubits

It is based on controlling the spins of electrons (holes) in semiconductor devices. Voltage and local magnetic fields are applied to control qubits. Quantum Computers based on spin qubits typically operate around 1 Kelvin.[\[10\]](#page-6-9) This technology is used by the company Intel [\[11\]](#page-6-10).

Superconducting Qubits

Superconducting qubits are based on superconducting circuit elements, which are macroscopic in size. Refer to Fig. [3](#page-1-1) for the transmon qubit by IBM, which is around 30 micrometers[\[12\]](#page-6-11). Therefore, it is different from other implementations which use microscopic systems such as atoms, or photons: the parameters (such as capacitance, inductance) are easier to change by setting the values of the circuit electrical elements. It operates around 20 mK. This technology is used by the companies IBM, Google, Alibaba, Rigetti, and Intel [\[11\]](#page-6-10).

FIG. 3: Four recent devices operating as superconducting transmon qubits and fabricated by IBM[\[13\]](#page-6-12)

Ion Trap Qubits

In this implementation, ions are confined and suspended in free space by using electromagnetic fields. Like NMR, it also works at the room temperature. See Fig. [4,](#page-2-0) where each ion trapped and suspended in the air are clearly visible. This technology is used by the companies IonQ, Honeywell and IQT [\[11\]](#page-6-10).

FIG. 4: Ion Trap Quantum Computer [\[1\]](#page-6-1)

Topological Qubits

Qubits in toplogical quantum computers are anyons, a quasi particle that occurs only in 2D systems. Topological qubits are supposedly more resistant to decoherence (how long a state can live) and noise. Operation temperature is around 0 Kelvin. Microsoft is the main company investing in this technology [\[11\]](#page-6-10); however, could not achieve to build a working quantum computer using topological qubits yet.

Photonic Qubits

Photons are used as qubits in this implementation. A single photon is prepared as "a superposition of two different colors"[\[14\]](#page-6-13). A possible mechanism to generate photonic qubits is presented in Fig. [5.](#page-2-1) It is possible to generate a photon of three different colors, which is then named as qurit. Optical instruments such as optical fibers and photon detectors are used for manipulation and detection. It operates at room temperature. This technology is used by the companies Psi Quantum, Xanadu and Orca [\[11\]](#page-6-10).

HISTORY

The history of quantum computing can be traced back to the introduction of quantum mechanics. However, the term started to be realized in a solid meaning in the 1980s. Yuri Manin is considered to be the one of the proposers of the idea of quantum computing[\[15\]](#page-6-14) for his paper Computable and noncomputable[\[16\]](#page-6-15).

After Manin, Feynmann comes with his talk[\[49\]](#page-7-0) on quantum computers in 1982, where he clarified the motivations for the need of a quantum computer. He stated that we do not understand everything in nature, and a universal computer may help us to do so. This computer should not imitate or give an approximate description of the nature. Instead, it should be an exact simulation, meaning it will "do exactly the same as nature". He defended that it is not possible to simulate quantum mechanical systems with classical computers, but it is only possible with a quantum mechanical computer.

Another important milestone in the history of quantum computing is Shor's Algorithm, which is introduced by Peter Shor in 1994.[\[17\]](#page-6-16) This algorithm is used for integer factorization. Theoretically, it could be used to break public-key cryptography schemes, and it was a powerful motivator for the development quantum computing algorithms back then.

Until the beginning of the 2000s, most of the work in the area of quantum computing were theoretical. Except for a few examples [\[5\]](#page-6-4) in the late 1990s, no working quantum computers were present. In 2000, a 5-qubit working NMR quantum computer is produced in the Technical University of Munich[\[18\]](#page-6-17), followed by a 7-qubit one in the Los Alamos National Laboratory[\[18\]](#page-6-17).

In 2001, IBM and Stanford University collaboration gave rise to the implementation of Shor's Algorithm on a 7-qubit NMR quantum computer. The number 15 is factorized by using the forenamed algorithm into 3 and 5.[\[18\]](#page-6-17) In 2012, number 21 is also factorized, which is the record for the largest number factored with a quantum computer by using Shor's algorithm after the factoriza-

FIG. 5: "The illustration shows the conversion of a photon of one frequency, or color, into a photon that is in a superposition of two colors..."[\[14\]](#page-6-13)

tion of 15. In the same year, the largest number factorized was 56153, which required 4 qubits and used a different algorithm, known as minimization algorithm.[\[19\]](#page-6-18)

In 2005, researchers from the University of Innsbruck, Austria created first qubyte, which is basically an entanglement of eight quantum particles[\[20\]](#page-6-19). In 2006, the collaboration of MIT, Institute for Quantum Computing, and Perimeter Institute gave rise to the 12-qubit system[\[21\]](#page-6-20).

In 2007, a Canadian based company D-Wave created a 16-qubit quantum computer.[\[22\]](#page-6-21) Although the company increased the number of qubits in the quantum computers in the following years $(2015: 1000+$ qubits $[23], 2020$ $[23], 2020$: 5000+ qubits[\[24\]](#page-6-23)), it was the highest number of qubits in a quantum computer.

In 2011, D-Wave announced the first commercial quantum computing, which is consisted of 128 qubits[\[25\]](#page-6-24). Google started to use D-Wave's quantum computers for tests in 2013. [\[22\]](#page-6-21). In 2015, Google reported that running a particular kind of algorithm on D-Wave's quantum computer (with 1000+ qubits) solved problems 100 million times faster than an Intel processor would solve [\[26\]](#page-6-25).

IBM initiated the first cloud-based quantum computer (of 5-qubits) in 2016 [\[27\]](#page-6-26). Later in 2017, it upgraded to 20-qubit cloud-based quantum computer[\[27\]](#page-6-26). After IBM's move in 2016, other companies also invested in cloud-based quantum computers. Fig. [6](#page-3-0) presents some of these companies and their technologies in a timeline.[\[27\]](#page-6-26)

FIG. 6: Cloud-based quantum computing systems[\[27\]](#page-6-26)

In 2019, Google announced that they have reached quantum supremacy. Briefly, quantum supremacy is a programmable quantum device solving a problem that no classical computer can solve, no matter how much time is given. Google's task handled the problem in 3 minutes. Same calculation is calculated to last 10 thousand years for a classical computer. [\[28\]](#page-6-27)

By the end of 2021, it is expected Chinese startup SpinQ to produce 2-qubit desktop quantum computer. The computer will be based on NMR, and will be working with a few drops of $C_2H_7O_3P$ liquid. Its market price is announced as \$5000.[\[29\]](#page-6-28)

	COMPUTING		
S.No	Description	Classical Computing	Quantum Computing
1	Information storage and representation	0 or 1	Qubit
$\overline{2}$	Delivery of information	Information can be copied without distributing.	Information cannot be copied distributing
3	Behaviour of information	Unidirectiona	Multidirectio nal
4	Security	Hacker can break into communicati on	Hacker cannot break into communicati on.
5	Noise Tolerance	Noisy channel can be used to deliver the information.	Noiseless channel is required.

FIG. 7: Comparison of Traditional and Quantum Computing[\[22\]](#page-6-21)

QUANTUM COMPUTER - CLASSICAL COMPUTER COMPARISON

There exists some fundamental differences between classical and quantum computers. The bit-qubit comparison is provided in the previous [section s](#page-0-1). Some of other differences are provided here briefly. See Fig. [7](#page-3-1) for an overall comparison in a chart.

Traditional computers are able to handle a single transaction at a time, whereas quantum computers are able to handle multiple transactions, making them potentially faster. Moreover, quantum computers are more secure. If one tries to to measure one parameter of a micro-particle, it will alter another parameter. So, it is impossible to hack without corrupting the state. Each attempt to spy a message alters the message.

There is an important theorem for quantum computing, known as "No-cloning theorem". It is related to the delivery of information, in other words, whether it is possible to copy information, or not. Theorem states that "it is impossible to create an independent and identical copy of an arbitrary unknown quantum state"[\[30\]](#page-7-1). So, information cannot be copied. This is an important consequence which distinguishes quantum computing from classical computing, where there is no limitation for copying bits in the latter one.

MAJOR CHALLENGES

Quantum computers are in general difficult to implement, and it is not even yet possible to create some types of qubits (like toplogical qubits) to use in quantum computing. The current working quantum computers mainly face these three problems: coherence, error correction, and maintaining low temperatures.

Coherence is basically the system's maintaining its own condition. Take a system in a state X. A coherent system stays in the state X if it is not otherwise commanded. Quantum computers, on the other hand, have low coherence time. In other words, the states of the qubits change by themselves after a while. Here, the time scale for decoherence is generally not days, or weeks, but seconds, or minutes in the best case. Until 2021, the longest coherence-time of a single qubit was 660 seconds, which has been recently increased to an hour. [\[31\]](#page-7-2). The main reason of decoherence is the sensitiveness of the quantum systems to the environment. Any noise, vibration, temperature fluctuation, electromagnetic wave, etc. affects the system to a great extent to break the coherence of the system. Overall, it is difficult for a quantum computer to preserve its state.

Due to the external effect of the environment on the quantum bits, it is almost impossible to create an errorfree quantum computer. Therefore, another challenge is to correct, or minimize these errors. This effort is pretty important, and led to a completely new area of research called quantum error correction. An approach is to adapt error correction methods that are readily available for bits to qubits; however, it is not straightforward due to the no-cloning theorem[\[32\]](#page-7-3), mentioned in the previous [section .](#page-3-2) A breakthrough approach came from Peter Shor (the creator of the Shor's algorithm) in 1995 [\[33\]](#page-7-4), where he suggested entangling quantum information across an expanded system of qubits is possible. It was important since it marked the era of practical quantum computers[\[32\]](#page-7-3), and many other approaches to quantum error correction has been developed in the last decades.

Another challenge is to maintain low temperatures of the quantum computers. As briefly introduced in the Physical Implementations [section ,](#page-1-2) many physical implementations of qubits require extremely low temperatures (close to 0 Kelvin). This is a problem in many ways: it is costly to reduce temperatures to that extent and keep it as cool as possible, it requires complex equipment to reach very low temperatures, it reduces the portability. At last, all these create opportunity for error. [\[4\]](#page-6-3)

Overall, the more qubits there are, the better performance a quantum computer delivers. Contrarily, as the number of qubits is increased, the more problems the system is likely to face with. Some of the major challenges (like low-temperature dependence) can be solved by changing the generation methods of qubits, but others

are not likely to be solved easily. The general approach is not to eliminate the errors completely, but reducingcontrolling them with quantum error correction.

WHEN (NOT) TO USE QUANTUM COMPUTERS

Quantum mechanics is probabilistic in its nature. Therefore, a single measurement is not what is generally wanted. Multiple measurements most likely to give better representations of the nature. Quantum Computers work on the same principle. They execute the same quantum algorithm over and over. The most likely result after these multiple runs is the solution.

For "very hard" problems, quantum computers offer a speed-up over classical computers. It is useful when running a quantum algorithm several times is still much faster than a single execution time of a classical computer. This is the case for very hard problems, sampling large datasets, optimization problems, machine learning, forecasting, etc.[\[34\]](#page-7-5) For other kinds of problems, it is not reasonable to use quantum computers.

As an example of very hard problems, the following two can be considered by the reader for clarification: simulation quantum chemistry, and factoring a number into its primes. The former one is a great fit for quantum computers because in quantum chemistry, every electronelectron interaction needs to be taken into account to simulate the system. Since at the atomic level the laws of quantum mechanics is dominant, a quantum computer that is not mimicking but naturally performing quantum mechanical properties can be very useful. The latter one, factoring a number, can be a bit controversial. As discussed in the preceding [section s](#page-2-2), factoring an integer like 15 into 3 and 5 probably do not require a quantum computer (recall that this is what IBM and Stanford University collaborated to achieve on a 7-qubit quantum computer in 2001). However, in some cases, a classical computer would take more than the age of the universe to factor a number[\[19\]](#page-6-18). Therefore, Shor's Algorithm to factor an integer into its prime by using a quantum computer can be an example of a quantum computational case where using a classical computer would be useless. On the other hand, a quantum computer would boost the performance time from a few ages of universe to a reasonable amount of time.

COMPANIES WORKING ON QUANTUM COMPUTING

There are currently many private companies involved in quantum computing research, along with some nations who pay close attention to the technological developments. Overall, it is possible to split the companies working on quantum computing into three categories: software & service providers, hardware & system providers, and end-to-end providers (both software and hardware). Below is a brief list of companies in each category, mostly gathered from Kukkuru's article [\[35\]](#page-7-6).

The major companies in the category of software and service providers are Zapata, Cambridge Quantum Computing, QC Ware, 1QBit, Riverlane, and QxBranch. QxBranch is acquired in 2019 by another company working on quantum computers: Rigetti. [\[36\]](#page-7-7). The others are mostly university-based companies. Zapata is spun out of Harvard[\[37\]](#page-7-8), Cambridge Quantum Computing conceived through University of Cambridge[\[38\]](#page-7-9), Riverlane also is a spin-out company from the University of Cambridge[\[39\]](#page-7-10). QC Ware is co-founded by three former researchers at NASA [\[40\]](#page-7-11)[\[41\]](#page-7-12). 1QBit is a Canada-based company with hardware partnerships with Microsoft, IBM, Fujitsu and D-Wave Systems[\[42\]](#page-7-13).

The major companies in the category of hardware and system provides are IonQ, QuTech, Intel, Quantum Circuits Inc. (QCI), BraneCell, and Tundra Systems Global LTD. QuTech is located in Delft University of Technology, in Netherlands. QCI is a Yale University startup company [\[43\]](#page-7-14). Others are American and English companies located in the U.S. and U.K. Among these companies, Intel is an important player since it is not solely a quantum computing company, but mainly a semiconductor company founded in 1968[\[44\]](#page-7-15).

The major companies that are working on both hardware and software are IBM, Google, Rigetti, Microsoft, Alibaba Group, D-Wave, Honeywell, Xanadu, and Qilimanjaro. In this category, most of the companies are not solely quantum computing companies, but big companies investing in many sectors. Google is interested in superconducting qubits, and its motivation for quantum computing is to use it in artificial intelligence. It has a research group called Google Quantum AI [\[45\]](#page-7-16). Microsoft is interested in topological qubits, which are supposedly more resistant to environmental effects, but could not be physically implemented yet[\[46\]](#page-7-17). Although it could not produce a working quantum computer using topological qubits, it has a cloud ecosystem called Azure Quantum. [\[47\]](#page-7-18). D-Wave is the owner of the first commercially available quantum computers, and Google (among others) used its quantum computers as mentioned in the preceding [section s](#page-2-2). IBM, rigetti and Honeywell are USbased companies, whereas Xanadu is Canada-based, like D-Wave. Qilimanjaro is a Spain-based company, whereas Alibaba is a Chinese multinational technology company. IBM, Rigetti, Alibaba, and Google are the ones that are investing heavily in superconducting qubits, whereas Honeywell is investing in ion trap qubits and Xanadu is investing in photonic qubits.

Overall, there is interest in quantum computing all over the world, with a worldwide market worth of 500M\$ by 2019. It is expected to reach 65B\$ by 2030[\[48\]](#page-7-19).

FIG. 8: Potential areas of use of quantum computing in near future. [\[51\]](#page-7-20)

POTENTIAL AREAS OF USE IN NEAR-FUTURE

Quantum computers are expected to be used in various areas in the near-future. The first sector that requires quantum computing is the aviation sector. Aviation requires it to reduce time spend on some processes and to overcome complexity. Jet software is too complicated for classical computers to handle and Lockheed Martin plans to use quantum computers to solve the issue [\[22\]](#page-6-21).

Machine Learning and Artificial Intelligence are some other areas that are going to benefit from the advancements in the quantum computing. Both requiring the use of huge amount of data, quantum computing can solve the processing time issues with the help of appropriate algorithms to run on quantum computers. An example for applying quantum computers in machine learning is discovering distant planets. There exists vast amount of data collected by spaceships and telescopes on Earth, and quantum computers may be used to process this data fast and accurately[\[22\]](#page-6-21).

Creating quantum simulations was an important motivation for the development of quantum computers [\[49\]](#page-7-0), and it is expected to be in the near future also. Other than sub-atomic research to understand the nature, quantum level simulations may be beneficial for health-related applications. They inherit the potential to understand how diseases develop, and develop more efficient drugs by analyzing DNA-sequencing data[\[22\]](#page-6-21). To understand biology and chemistry, which are highly dependent on microscopic interactions of molecules, quantum computers are expected to be very useful.

Precision forecasting is another area quantum computers can be used excessively. Especially the automative sector is currently investing heavily in quantum computing for the self-driving technology. Companies like Google, Volkswagen, Daimler, Ford, and many more are taking steps into quantum computing to reduce traffic congestion, for route optimization, minimal waiting time, etc. There are many important collaborations taking place, such as Google producing qubit-used chips for Daimler, and Ford teaming up with NASA[\[50\]](#page-7-21).

QUANTUM COMPUTING PATENT FAMILIES BY CATEGORY AND PUBLICATION YEAR

FIG. 9: Quantum Computing patents over years[\[1\]](#page-6-1)

CONCLUSIONS

Starting with the introduction of quantum theory in 1927, and the concept of quantum computers in the 1980s, the research on and the application of quantum computing grew exponentially. As one gets closer to today in the timeline, it is possible to observe the developments are getting faster and more advanced. Fig. [1](#page-0-0) is a compact timeline chart showing the exponential growth of the research area of quantum computing. Fig. [9](#page-6-29) shows quantum computing patents over years (starting from 1998), which can also be considered as an indicator of how quantum computing became a hot topic in the recent years.

Quantum computing promises radical improvements in various areas. Throughout the article, some of these potential areas of improvement tried to be touched upon. The reader can refer especially to the preceding [sec](#page-5-0)[tion .](#page-5-0) Many companies and nations are investing in it since it will have an important role in the future technology. Before 2050, it is expected that today's widely used encryption techniques such as RSA will be destroyed within days to hours with the advancement of Quantum Computing[\[52\]](#page-7-22). It is likely that we will need new methods and technologies optimized for the quantum computers in many fields in the upcoming years. Meanwhile, further research to make quantum computers more coherent, less erroneous and portable is likely to be conducted.

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